Problem 1: Solve the inequation

$$\sqrt{\left(\log_2 x\right)^2+4\log_2 x+6}-\sqrt{\left(\log_2 x\right)^2-6\log_2 x+11}>\sqrt{3-\log_2 x}-\sqrt{\log_2 x+2}.$$

Solve:

Condition: $-2<\log_2 x<3 \Leftrightarrow \frac{1}{4}<x<8 \ (*)$.

From $\sqrt{\left(\log_2 x\right)^2+4\log_2 x+6}-\sqrt{\left(\log_2 x\right)^2-6\log_2 x+11}>\sqrt{3-\log_2 x}-\sqrt{\log_2 x-2}$

deduce $\sqrt{\left(\log_2 x+2\right)^2+2}+\sqrt{\log_2 x+2}>\sqrt{\left(3-\log_2 x\right)^2+2}+\sqrt{3-\log_2 x}$ (1)

Consider function $f\left(u\right)=\sqrt{u^2+2}+\sqrt{u}$ has $f'\left(u\right)=\frac{u}{\sqrt{u^2+2}}+\frac{1}{2\sqrt{u}}>0,\ \forall u>0$. Hence f interesting over $\left(0;+\infty\right)$. Thus from (1) deduce:

$$\log_2 x+2>3-\log_2 x \Leftrightarrow \log_2 x>1 \Leftrightarrow x>2.$$

Combine (*), hence inequation root: $2<x<8$.

Problem 2: Finds m such that inequation $x\sqrt{x}+\sqrt{x+9}\ge m\left(\sqrt{16-x}+\sqrt{15-x}\right)$ has roots all in $\left[0;15\right]$.

Solve:

Condition: $x\in\left[0;15\right]$.

From $x\sqrt{x}+\sqrt{x+9}\ge m\left(\sqrt{16-x}+\sqrt{15-x}\right)$ deduce

$$\left(x\sqrt{x}+\sqrt{x+9}\right)\left(\sqrt{16-x}-\sqrt{15-x}\right)\ge m$$

Consider functions $f\left(x\right)=x\sqrt{x}+\sqrt{x+9},\ g\left(x\right)=\sqrt{16-x}-\sqrt{15-x}$

$$f'\left(x\right)=\frac{3}{2}\sqrt{x}+\frac{1}{2\sqrt{x+9}}>0,\ g'\left(x\right)=\frac{\sqrt{16-x}-\sqrt{15-x}}{\sqrt{16-x}.\sqrt{15-x}}>0$$

all $x\in\left[0;15\right]$.

Hence $h\left(x\right)=f\left(x\right)g\left(x\right)$ interesting over $\left[0;15\right]$ and h continuous over $\left[0;15\right]$.

Thus: $m\le\min\limits_{\left[0;15\right]}h\left(x\right)=3\left(4-\sqrt{15}\right)$.