Problem: Determine the equation of the line Δ through the point $A(1;2)$ such that sum distance from point $B(3;4)$ and $C(0;-2)$ to Δ have maximum value.

Solve:

The equation of the line Δ through the point $A(1;2)$:

$$a.(x-1)+b.(y-2)=0 \ \left(a^2+b^2>0\right)$$

$$\Leftrightarrow ax+by-a-2b=0$$

$$dist(B,\Delta)+dist(C,\Delta)=\frac{|2a+2b|}{\sqrt{a^2+b^2}}+\frac{|a+4b|}{\sqrt{a^2+b^2}}$$

Follow inequalities $|\alpha+\beta|\le|\alpha|+|\beta|$ and Bunhiacopxki then

$$dist(B,\Delta)+dist(C,\Delta)=\frac{1}{\sqrt{a^2+b^2}}\left(|2a+2b|+|a+4b|\right)$$

$$\le\frac{1}{\sqrt{a^2+b^2}}\left(3|a|+6|b|\right)\le\frac{1}{\sqrt{a^2+b^2}}.\sqrt{3^2+6^2}.\sqrt{a^2+b^2}=3\sqrt{5}.$$

Thus $\max\left(dist(B,\Delta)+dist(C,\Delta)\right)=3\sqrt{5}$ when $ab>0$ and $\frac{|a|}{|b|}=\frac{1}{2}$. Choose $b=2$ then $a=1$.

Hence $\Delta: x+2y-5=0$.