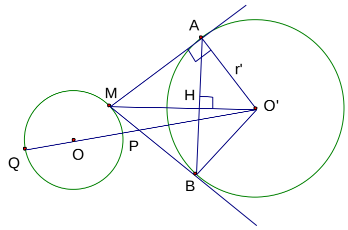
Problem 1: Give two circle (O,r) and (O',r') outside. Let a point M belong circle (O,r) and draw two tangent line MA, MB to circle (O',r') (A, B belong (O',r')). Finds position of point M such that minimum and maximum values line segment AB.
Solve:



Let H is midpoint of line segment AB. In the quad triangle $O'AM$:

$$O'M.AH = AM.AO' \Rightarrow AB = \frac{2AM.AO'}{O'M} = \frac{2\sqrt{O'M^2 - AO'^2}.AO'}{O'M}$$

$$= 2r'\sqrt{1 - \frac{r'^2}{O'M^2}}$$

Calls $P = OO' \cap (O,r)$, $Q = OO' \cap (O,r)$ then $O'P \le O'M \le O'Q$. Hence:

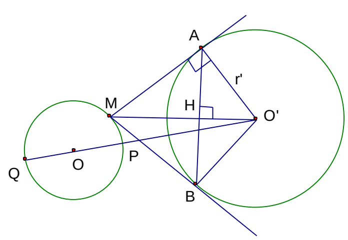
$$2r'\sqrt{1-\frac{r'^2}{O'P^2}} \le AB = 2r'\sqrt{1-\frac{r'^2}{O'M^2}} \le 2r'\sqrt{1-\frac{r'^2}{O'Q^2}}.$$

If $P \equiv Q$ then $AB = 2r'\sqrt{1-\frac{r'^2}{O'P^2}} = 2r'\sqrt{1-\frac{r'^2}{OO'^2 - r^2}}$

Hence $AB_{\min} = 2r'\sqrt{1-\frac{r'^2}{O'P^2}}$ and $AB_{\max} = 2r'\sqrt{1-\frac{r'^2}{O'Q^2}}$.

Problem 2: Give four a,b,c,d reals satisfy an equations $a^2+b^2=1$ and $(c-4)^2+(d-4)^2=4$. Finds minimum and maximum values $F = 1 - \frac{4}{33-8(a+b)}$.

Solve:



Follow problem 1, consider an circles $(C_1): x^2+y^2=1$ and $(C_2): (x-4)^2+(y-4)^2=4$ outside.
Hence $M(a;b) \in (C_1)$. Draw two tangents line MA, MB to circle (I_2, r_2) (A, B belong (I_2, r_2)). Thus $F = 1 - \frac{4}{33-8(a+b)} = 1 - \frac{4}{(a-4)^2+(b-4)^2} = \frac{AB^2}{16}$.

Circle (C_1) has center $I_1(0;0)$, radius $r_1 = 1$ and circle (C_2) has center $I_2(4;4)$, radius $r_2 = 2$.

Follow problem 1, $AB_{\min} = 2r_2\sqrt{1-\frac{r_2^2}{I_2P^2}}$ and $AB_{\max} = 2r_2\sqrt{1-\frac{r_2^2}{I_2Q^2}}$ with

$P = I_1I_2 \cap (I_1, r_1)$, $Q = I_1I_2 \cap (I_1, r_1)$.

Calculates $AB_{\min} = 4\sqrt{2}-1$ and $AB_{\max} = 4\sqrt{2}+1$.

Thus $\max F = \frac{AB_{\max}^2}{16} = \frac{33+8\sqrt{2}}{16}$ and $\min F = \frac{AB_{\min}^2}{16} = \frac{33-8\sqrt{2}}{16}$ or have inquality

$$\frac{33-4\sqrt{2}}{16} \le 1-\frac{4}{33-8(a+b)} \le \frac{33+4\sqrt{2}}{16}.$$