

THE IMPROVED QUADRATIC FORMULA IN GRAPHIC FORM

(Authored by Nghi H Nguyen – Updated on April 15, 2020)

There are so far 8 common methods to solve a quadratic equation in standard form: $ax^2 + bx + c = 0$. They are: quadratic formula, completing the square, factoring FOIL method, graphing, the Bluma Method, the Diagonal Sum Method, the factoring AC method, and the new “Transforming Method”, recently introduced on Google, Bing, and Yahoo Search.

If the given quadratic equation can be factored, the new **Transforming Method** may be the best solving method since it can immediately obtain the 2 real roots without factoring by grouping and solving binomials.

In case the given quadratic equation can't be factored, the **quadratic formula** would be the best choice to solve it. There is an improved quadratic formula, called: “**The improved quadratic formula in graphic form**”, that is simpler and easier to remember than the classical formula.

The quadratic function in intercept form

The graph of the quadratic function in standard form $f(x) = ax^2 + bx + c$ is a **parabola** that may intercept the x-axis at 2 points, unique point, or no point at all. This means a quadratic equation $f(x) = ax^2 + bx + c = 0$ may have 2 real roots, one double real root, or no real roots at all (complex roots).

We can write $f(x)$ in the form: $y = a(x^2 + b/a x + c/a)$ **(1)**

Recall the development of the quadratic formula:

$$(x^2 + bx/a + b^2/4a^2 - b^2/4a^2 + c/a = 0$$

$$(x + b/2a)^2 - (b^2 - 4ac)/4a^2 = 0$$

$$(x + b/2a)^2 - d^2/4a^2 = 0. \quad (\text{Call } d^2 = b^2 - 4ac)$$

$$(x + b/2a + d/2a)(x + b/2a - d/2a) \quad \mathbf{(2)}$$

Replace this expression (2) into the equation (1), we get the quadratic function $f(x)$ written in intercept form:

$$f(x) = a(x - x_1)(x - x_2) \quad (x_1 \text{ and } x_2 \text{ are the 2 x-intercepts, or roots of } f(x) = 0)$$

$$\mathbf{f(x) = a(x + b/2a + d/2a)(x + b/2a - d/2a) \quad (3)}$$

The improved quadratic formula in graphic form.

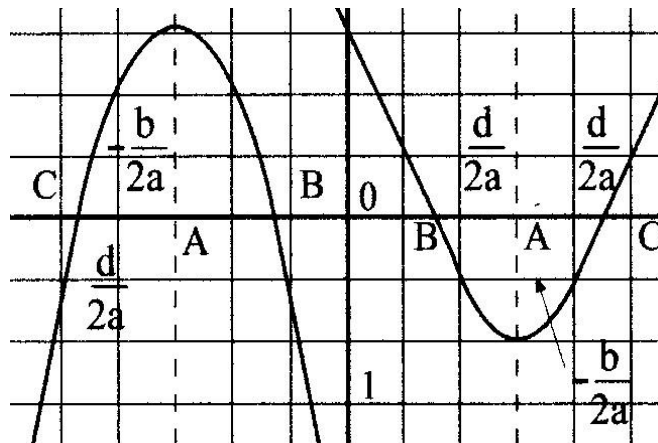
From the equation (3), we deduce the formula:

$$x = -b/2a \pm d/2a \quad (4)$$

In this formula:

- The quantity $(-b/2a)$ represents the x-coordinate of the parabola axis.
- The 2 quantities $(d/2a)$ and $(-d/2a)$ represent the 2 distances AB and AC from the parabola axis to the two x-intercepts (real roots) of the parabola.
- The quantity d can be zero, a number, or imaginary.
- If $d = 0$: there is a double root at $x = -b/2a$.
- If d is a number (real or radical), there are two x-intercepts, meaning two real roots.
- If d is imaginary, there are no real roots. The parabola doesn't intercept the x-axis.
- The unknown quantity d can be computed from the constants a, b, c by this relation (5):

$$d^2 = b^2 - 4ac \quad (5)$$



We can easily obtain this relation by writing that the product ($x_1 \cdot x_2$) of the 2 real roots is equal to (c/a) .

$$(-b/2a + d/2a)(-b/2a - d/2a) = c/a$$

$$b^2 - d^2 = 4ac \rightarrow d^2 = b^2 - 4ac$$

To solve a quadratic equation, first find the quantity **d** by the **relation (5)**, then find the 2 real roots by the **formula (4)**.

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Advantage of the improved formula in graphic form

This new quadratic formula in graphic form is simpler and easier to remember than the classic formula since students can relate it to the x-intercepts of the parabola graph. In addition, the two quantities **(d/2a)** and **(-d/2a)** make more sense about distance than the classic quantity: $\sqrt{(b^2 - 4ac)}$.

In addition, many quadratic equations has 2 real roots in the form of 2 fractions. If we use the quadratic formula, the computed 2 real roots will be in decimals, while the correct answers are 2 fractions.

Example. Solve $8x^2 - 22x - 13 = 0$

Solution. If we use quadratic formula and compute by calculator, the 2 real roots are:

$$x_1 = -0.5 \text{ and } x_2 = 3.25.$$

If we use the Quadratic Formula in Graphic Form, the correct answers will be:

$$x_1 = -1/2, \text{ and } x_2 = 13/4$$

Example. Solve $24x^2 + 59x + 36 = 0$

Solution. Use calculator with quadratic formula: $x_1 = -1.33$ and $x_2 = -1.125$

Use Quadratic Formula in Graphic Form, the correct answers are: $x_1 = -4/3$ and $x_2 = -9/8$

Examples of solving quadratic equations by the Quadratic Formula in Graphic Form.

Example 1. Solve: $4x^2 - 12x + 9 = 0.$

Solution. First, find d by the relation (5):

$$d^2 = b^2 - 4ac = 144 - 144 = 0 \rightarrow d = 0$$

The equation has double root at $x = -b/2a = 12/8 = 3/2$.

Example 2. Solve: $2x^2 - 3x + 7 = 0$

Solution. First find d^2 by the relation (5).

$$d^2 = 9 - 56 = -47.$$

d is imaginary, there are no real roots.

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Example 3. Solve: $3x^2 + 16x - 12 = 0$.

Solution. $d^2 = 256 + 144 = 400 = 20^2$

Next, find the 2 real roots by the formula (4).

$$x_1 = -16/6 + 20/6 = 4/6 = 2/3$$

$$x_2 = -16/6 - 20/6 = -36/6 = -6.$$

Example 4. Solve: $7x^2 + 18x - 25 = 0$.

Solution. $d^2 = 324 + 700 = 1024 = 32^2$

$$x_1 = -18/14 + 32/14 = 14/14 = 1$$

$$x_2 = -18/14 - 32/14 = -50/14 = -25/7$$

Example 5. Solve: $2x^2 + 12x + 17 = 0$

Solution. $d^2 = 144 - 136 = 8 \rightarrow d = 2.83$

$$x_1 = -12/4 + 2.83/4 = -9.17/4 = -2.29$$

$$x_2 = -12/4 - 2.83/4 = -14.83/4 = -3.70$$

Example 6. Solve: $5x^2 - 10x - 3 = 0$.

Solution. $d^2 = 100 + 60 = 160 \rightarrow d = 12.65$

$$x_1 = 10/10 + 12.65/10 = 22.65/10 = 2.26$$

$$x_2 = 10/10 - 12.65/10 = -2.65/10 = -0.26$$

NOTE. When the given quadratic equation can be factored, its 2 real roots are usually in the form of two fractions. The quantity d^2 should be a perfect square and d should be a **whole number**. So, students are advised to proceed solving by the formula in graphic form, mentally or by using calculators, through 2 steps. First step, find d by the relation (5). Second step, **algebraically** calculate the 2 real roots by the Formula (4). In case d is a whole number, make

sure that the 2 real roots (answers) be in the form of 2 fractions and not in decimals as given by calculators.

[This article was written by Nghi H. Nguyen, author of the new Transforming Method (Google, Bing, or Yahoo Search) for solving quadratic equations – Updated on April 15, 2020]