SOLVING QUADRATIC EQUATIONS BY THE FORMULA IN GRAPHIC FORM

(By Nghi H Nguyen)

There are many methods to solve a quadratic equation in standard form: $ax^2 + bx + c = 0$.

If the given equation can be factored, the two best methods to solve it are "the factoring ac method" (You Tube) and the new Diagonal Sum Method. In case the given quadratic equation can't be factored, the quadratic formula would be the best way to solve it.

An improved quadratic formula, called: "The quadratic formula in graphic form", has been presented in the book titled: "New methods for solving quadratic equations and inequalities" (Amazon e-book 2010).

The graph of the quadratic function $f(x) = ax^2 + bx + c$ is a parabola that may intercept the x-axis at 2 points, unique point, or no point at all. This means a quadratic equation $f(x) = ax^2 + bx + c = 0$ may have 2 real roots, one double real root, or no real roots at all (complex roots).

Given a quadratic function in standard form $f(x) = ax^2 + bx + c$ and its parabola graph drawn on a coordinate system Ox,Oy. Suppose A is the x-intercept of the parabola axis, and suppose the parabola intercepts the x-axis at 2 points B and C.

The quadratic formula in graphic form gives the 2 real roots x1 and x2 by this formula:

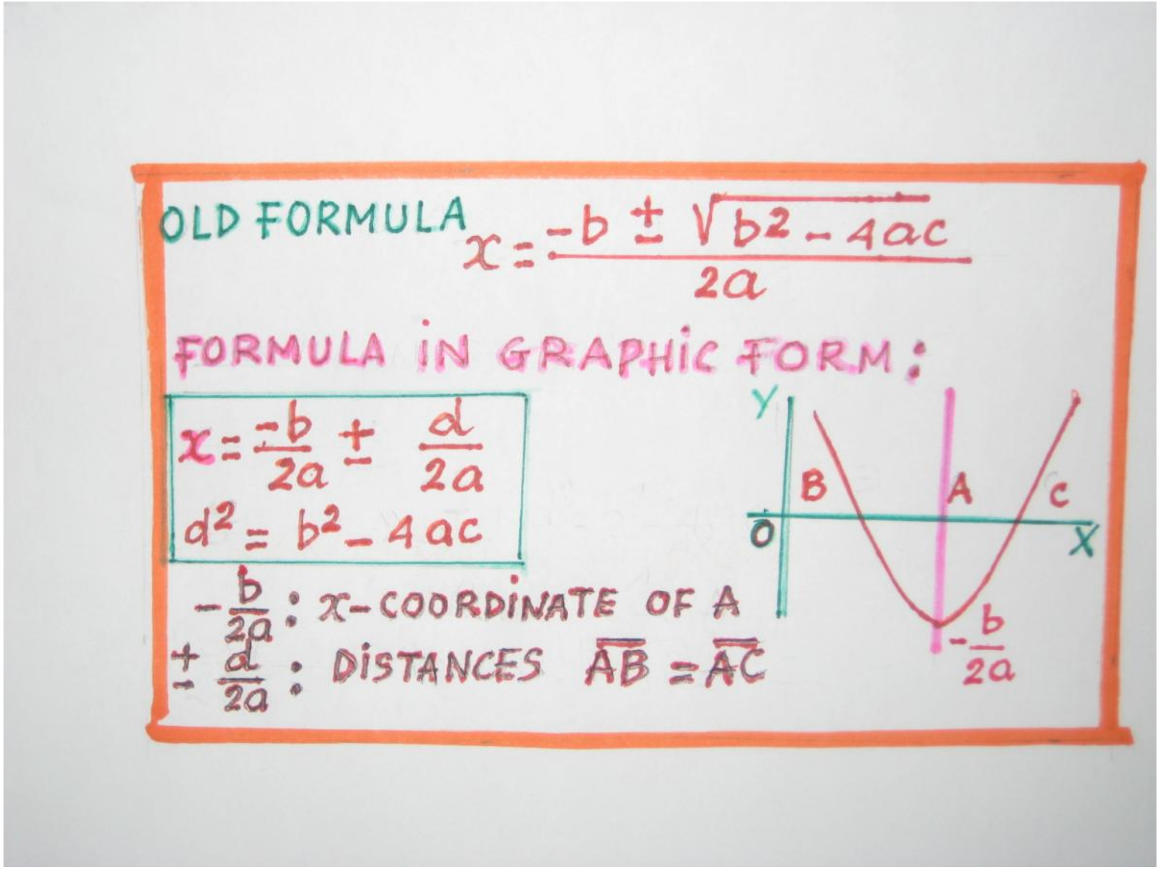
$$x1 = -b/2a + d/2a \quad \text{and} \quad x2 = -b/2a - d/2a \qquad (1)$$

In this formula:

- The quantity **(–b/2a)** represents the x-coordinate of the parabola axis.
- The 2 quantities **(d/2a)** and **(-d/2a)** represent the 2 distances AB and AC from the parabola axis to the two x-intercepts (real roots) of the parabola.
- The quantity **d** can be zero, a number, or imaginary.
- If d = 0: there is a double root at x = -b/2a.
- If d is a number (real or radical), there are two x-intercepts, meaning two real roots.
- If d is imaginary, there are no real roots.

- The unknown quantity **d** can be computed from the constants a, b, c by this relation (2):

$$d^2 = b^2 - 4ac \qquad (2)$$



We can easily obtain this relation by writing that the product (x1.x2) of the 2 real roots is equal to (c/a).

(-b/2a + d/2a)(-b/2a – d/2a) = c/a

b^2 – d^2 = 4ac → d^2 = b^2 – 4ac

To solve a quadratic equation, first find the quantity d by the relation (2), then find the 2 real roots by the formula (1).

REMARK. This improved quadratic formula is simpler and easier to remember than the classic formula since students can relate it to the x-intercepts of the parabola graph. In addition, the two quantities (d/2a) and (-d/2a) make more sense about distance than the classic quantity “square root of b^2 – 4ac”.

Examples of solving quadratic equations by the quadratic formula in graphic form.

Example 1. Solve: $4x^2 - 12x + 9 = 0$.
Solution. First, find d by the relation (2):
$d^2 = b^2 - 4ac = 144 - 144 = 0 \rightarrow d = 0$
The equation has double root at $x = -b/2a = 12/8 = 3/2$.

Example 2. Solve: $2x^2 - 3x + 7 = 0$
Solution. First find d^2 by the relation (2).
$d^2 = 9 - 56 = -47$.
d is imaginary, there are no real roots.

Example 3. Solve: $3x^2 + 16x - 12 = 0$.
Solution. $d^2 = 256 + 144 = 400 = (20)^2$
Next, find the 2 real roots by the formula (1).
$x1 = -16/6 + 20/6 = 4/6 = 2/3$
$x2 = -16/6 - 20/6 = -36/6 = -6$.

Example 4. Solve: $7x^2 + 18x - 25 = 0$.
Solution. $d^2 = 324 + 700 = 1024 = (32)^2$
$x1 = -18/14 + 32/14 = 14/14 = 1$
$x2 = -18/14 - 32/14 = -50/14 = -25/7$

Example 5. Solve : $2x^2 + 12x + 17 = 0$
Solution. $d^2 = 144 - 136 = 8 \rightarrow d = 2.83$
$x1 = -12/4 + 2.83/4 = -9.17/4 = -2.29$
$x2 = -12/4 - 2.83/4 = -14.83/4 = -3.70$

Example 6. Solve: $5x^2 - 10x - 3 = 0$.
Solution. $d^2 = 100 + 60 = 160 \rightarrow d = 12.65$
$x1 = 10/10 + 12.65/10 = 22.65/10 = 2.26$
$x2 = 10/10 - 12.65/10 = -2.65 /10 = -0.26$

NOTE. When the given quadratic equation can be factored, its 2 real roots are usually in the form of two fractions. The quantity d^2 should be a perfect square and **d** should be a **whole number**. So, students are advised to proceed solving by the formula in graphic form, mentally or by using calculators, through 2 steps. First step, find d by the relation (2). Second step, **algebraically** calculate the 2 real roots by the formula (1). In case d is a whole number, make sure that the 2 real roots (answers) be in the form of 2 fractions and not in decimals.

(This article was written by Nghi H. Nguyen, co-author of the new Diagonal Sum Method for solving quadratic equations)