SOLVING QUADRATIC EQUATIONS - COMPARE THE FACTORING "AC METHOD" AND THE NEW TRANSFORMING METHOD

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GENERALITIES.

When a given quadratic equation can be factored, there are 2 best methods to solve it: the "AC factoring Method" (YouTube.com) and the newly introduced Transforming Method (Google or Yahoo Search). This article explains, in general, these two methods, and then compares them through specific examples.

THE FACTORING AC METHOD

This method only applies to quadratic equations that can be factored. It is also called "the Box Method" with a little variation (You Tube). Its concept is to make the given quadratic equation be factored into 2 binomials in x by replacing the term (bx) by 2 terms (b1x) and (b2x) that satisfy the 2 conditions:
1) The product b1*b2 = a*c; and 2) The sum (b1 + b2) = b.

<u>Example 1.</u> Solve: 5x^2 + 6x – 8 = 0

Solution. Find 2 numbers that the product is (a*c = -40) and the sum is b = 6. Proceeding: [(-1, 40),(1, -40),(-2, 20),(2, -20),(-4, 10) OK]. Next, substitute in the equation the term (6x) by the 2 terms (-4x) and (10x) and then factor by grouping:
5x^2 – 4x + 10x – 8 = 5x(x + 2) – 4(x + 2) = (x + 2)(5x – 4) = 0
Next, solve the 2 binomials for x:
(x + 2) = 0 → x = -2, and (5x – 4) = 0 → x = 4/5

THE NEW TRANSFORMING METHOD

This method uses 2 features in its solving process.

1. **Recall The Rule of Signs for Real Roots of a quadratic equation.**

*If a and c have opposite signs, the 2 real roots have opposite signs.
Example: the equation: 7x^2 – 5x - 12 = 0 has 2 real roots (-1) and (12/7) with different signs

* If a and c have the same sign, both real roots have same sign.
- When a and b have same sign, both real roots are negative.
- When a and b have opposite signs, both real roots are positive.

Example: the equation: 6x^2 + 13x + 7 = 0 has 2 real roots (-1) and (-7/6), both negative
Example: The equation: 5x^2 - 9x + 4 = 0 has 2 real roots (1) and (4/5), both positive

2. **The Diagonal Sum Method to solve quadratic equation type x^2 + bx + c = 0, (a = 1).**

In this case, solving results in finding 2 numbers knowing their sum (**-b**) and their product **c**. This method proceeds by composing factor pairs of **c**, following these 3 Tips.

TIP 1. When roots have different signs, compose factor pairs of **c** with all first numbers being **negative**
Example 1. Solve: x^2 – 31x - 102 = 0.
Roots have different signs. Compose factor pairs of c = -102 with all first numbers being negative. Proceeding: (-1, 102), (-2, 51), (-3, 34). OK. This sum is 34 – 3 = 31 = -b. The 2 real roots are -3 and 34.

Important Note. If you can't find the pair whose sum equals to (-b), or b, then this quadratic equation can't be factored, and you should probably use the quadratic formula to solve it.

TIP 2. When both roots are positive, compose factor pairs of c with all positive numbers.
Example 2. Solve: x^2 - 28x + 96 = 0.
Both roots are positive. Compose factor pairs of c = 96 with all positive numbers. Proceeding: (1, 96)(2, 48)(3, 32)(4, 24). This last sum is 4 + 24 = 28 = -b. The 2 real roots are 4 and 28.

TIP 3. When both roots are negative, compose factor pairs of c with all negative numbers.
Example 3. Solve: x^2 + 39x + 108 = 0.
Both roots are negative. Compose factor pairs of c = 108 with all negative numbers. Proceeding: (-1, -108)(-2, -54)(-3, -36). This last sum is -3 - 36 = -39 = -b. The 2 real roots are: -3 and -36.

SOLVING STEPS OF THE TRANSFORMNG METHOD

In general, this new method proceeds through 3 steps.

STEP 1. Transformation of equation, from standard form ax^2 + bx + c = 0 (1) to simplified form, with a = 1 and with the constant C = a*c. The transformed form is: x^2 + bx + a*c = 0. (2).

STEP 2. Solve the transformed equation (2) by the Diagonal Sum Method that immediately obtains the 2 real roots: **y1** and **y2.**

STEP 3. Divide both y1 and y2 by the coefficient **a** to get the 2 real roots of the original equation (1): **x1** = y1/a, and **x2** = y2/a.

COMPARE THE AC METHOD TO THE TRANSFORMING METHOD THROUGH EXAMPLES

Example 4**.** Solve: 8x^2 – 22x – 13 = 0. (1)

The Transforming Method. Transformed equation: x^2 – 22x – 104 = 0 (2). Roots have different signs. Solve the transformed equation (2) by composing factor pairs of a*c = -104. Proceeding: (-1, 104)(-2, 53)(-4, 26). This last sum is -4 + 26 = 22 = -b. The real roots are: **y1** = 4 and **y2** = 26. Then the 2 real roots of the original equation are: **x1** = y1/8 = -4/8 = -1/2, and **x2** = y2/8 = 26/8 = 13/4.

The AC Method. Find 2 numbers, that their product is ac = -104, and their sum is -22. Proceeding: (-1, 104),(1, -104),(-2, 52),(2, -52), (-4, 26),(4, -26). Then, b1 = 4 and b2 = -26. Replace in the equation the term (-22x) by the 2 terms (4x) and (-26x) then, factor by grouping:

8x^2 + 4x – 26x – 13 = 4x(2x + 1) – 13(2x +1) = (2x + 1) (4x – 13) = 0.

Next, solve the 2 binomials:

(2x + 1) = 0 → x = -1/2, and (4x – 13) = 0 → x = 13/4.

Example 5. Solve: 12x^2 + 5x – 72 = 0. (1)

Transforming Method. Transformed equation: x^2 + 5x – 864 = 0 (2). Roots have different signs. Compose factor pairs of a*c = -864. Since b = 5 is too small as compared to a*c = -864, start composing from the middle of the factor chain. Proceeding: .....(-18, 48)(-24, 36)(-32, 27). This last sum is: -32 + 27 = -5 = -b. The 2 real roots of (2) are **y1** = -32, and **y2** = 27.

Back to the original equation (1), the 2 real roots are: **x1** = y1/12 = -32/12 = -8/3, and **x2** = y2/12 = 27/12 = 9/4.

The AC Method. In this case, solving becomes inconvenient because the product ac is a large number. Find 2 numbers: product ac = -864, and sum b = 5. Proceeding: [(-1, 864),(1, -864),(-2, 432),(2, -432),(-3, 288),…(-18, 48),(18, -48),(-24, 36),(24, -36),(- 32, 27),(32, -27) OK]. Next, replace the term 5x by the 2 terms -27x and 32x, then factor by grouping.

12x^2 – 27x + 32x – 72 = 3x(4x – 9) + 8(4x - 9) = (4x – 9)(3x + 8) = 0.

Next, solve the 2 binomials:

(4x – 9) = 0 → x = 9/4, and (3x + 8) = 0 → x = -8/3.

Example 6**.** Solve: 24x^2 + 59x + 36 = 0.

Transforming Method. Transformed equation: x^2 + 59x + 864 = 0 (2). Both roots are negative. Compose factor pairs of a*c = 864 with all negative numbers. Start composing from the middle of the chain. Proceeding: ……..(-18, -48)(-24, -36)(-27, -32). This last sum is -59 = -b. The 2 real roots of (2) are: **y1** = -27, and **y2** = -32. Back to the original (1), the 2 real roots are: **x1** = -27/24 = -9/8, and **x2** = -32/24 = -4/3.

The AC Method. Find 2 numbers: product ac = 864; sum b = 59. Proceeding [(-1, -864)(1, 864),(-2 , -432),(2, 432),(-4, -216),(4, 216), …………(-18, - 48)(18, 48)(-24, -36) (24, 36),(-27, -32)(27, 32)]. Since the product ac = 864 is too large, the proceeding takes too much time to complete.

Next, replace term (59x) by the 2 term(27x) and (32x), then factor by grouping:

24x^2 + 32x + 27x + 36 = 8x(3x + 4) + 9(3x + 4) = (3x + 4)(8x + 9) = 0

(3x + 4) = 0 → x = -4/3, and (8x + 9) = 0 → x = -9/8.

CONCLUSION AND REMARKS.

Both methods deserve to be studied because they provide students with opportunities to improve math skills and logical thinking that are the ultimate goals of learning math. However, the Transforming Method looks simpler and faster. In any case, it can immediately obtains the 2 real roots without factoring by grouping, and solving the 2 binomials.

The strong points of the new Transforming Method are: simple, fast, systematic, no guessing, and no solving binomials.

References:

- Solving quadratic equations by the Diagonal Sum Method (Google and Yahoo Search)
- The Bluma Method (Google Search)
- Factoring Trinomials – Simplified AC Method (www.200clicks.com/mathhelp)
- AC Method for Factoring – Regent Exam Prep Center (Google or Yahoo Search)