

SOLVING QUADRATIC EQUATIONS BY THE NEW “TRANSFORMING METHOD”

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There are so far 8 common methods to solve quadratic equations in standard form $ax^2 + bx + c = 0$. They are: graphing, completing the squares, factoring FOIL method, quadratic formula, the Bluma Method, the Diagonal Sum Method, the popular factoring AC Method, and the new Transforming Method that was recently introduced on Google, Yahoo, Bing Search.

This new method is fast, effective, systematic, no guessing, and it is applicable whenever the quadratic equation can be factored. It can obtain the 2 real roots without lengthy factoring by grouping, and without solving the 2 binomials.

This new method uses in its solving process three features:

1. The Rule of Signs For Real Roots of a quadratic equation that shows the signs (- or +) of the 2 real roots in order to select a better solving approach.
2. CASE 1. Solving quadratic equations type $x^2 + bx + c = 0$, with $a = 1$
3. CASE 2. Transformation of a quadratic equation in standard form $ax^2 + bx + c = 0$ (1) into a simplified quadratic equation, with $a = 1$, for a faster solving approach.

RECALL THE RULE OF SIGNS FOR REAL ROOTS OF A QUADRATIC EQUATION

- If **a** and **c** have opposite signs ($ac < 0$), the 2 real roots have opposite signs.

Example. The equation $x^2 - 8x - 9 = 0$ has 2 real roots with opposite signs: -1 and 9.

- If **a** and **c** have same sign ($ac > 0$), the 2 real roots have same sign.

Example. The equation $5x^2 - 14x + 9 = 0$ has 2 real roots both positive: 1 and 9/5.

Example. The equation $7x^2 + 8x + 1 = 0$ has 2 negative real roots: (-1) and (-1/7)

CASE 1. SOLVING QUADRATIC EQUATIONS TYPE: $x^2 + bx + c = 0$, with $a = 1$

In this case, solving results in finding 2 numbers knowing their sum (-b) and their product (c).

A. Simple quadratic equations.

When c is a small number, we can directly get the 2 real roots by guessing, or by applying some shortcut methods.

Example 1. Solve: $y = x^2 - 5x - 14 = 0$.

Solution. By guessing, we get the 2 real roots: -2 and 7. Their product is ($c = -14$), and their sum is ($-b = 5$).

Note. We can also factor the quadratic equation in the form: $y = (x + a)(x + b)$
There are many videos on YouTube that show students how to find the numbers (a) and (b). Finally, we solve the 2 binomials to get the 2 real roots.

Example 2. Solve: $y = x^2 - 11x + 18 = 0$.

Solution. $y = (x + a)(x + b) = 0$

Any method on YouTube would give $y = (x - 2)(x - 9)$. After solving the 2 binomials, we get the 2 real roots (2), and (9)

B. Complex quadratic equations.

When c , or the product (ac), are large numbers, guessing and simplified methods aren't effectively working. The Transforming Method proceeds by composing factor pairs of (c), then, by finding the pair whose sum is equal to ($-b$).

While composing factor pairs of (c), it is advised that the 2 following TIPS be followed:

TIP 1. When roots have opposite signs ($ac < 0$). Compose the factor pairs of c with all first numbers of the pairs being negative. Stop composing when you find the pair whose sum is equal to (**b**), or (**-b**). If you can't find it, then the equation can't be factored, and you probably should use the quadratic formula to solve it.

Example 3. Solve: $x^2 - 11x - 102 = 0$.

Solution. The Rule of Signs indicates roots have opposite signs ($ac < 0$).

First, compose factor pairs of ($c = -102$) with all first numbers being negative.

Proceeding: (-2, 51), (-3, 34), (-6, 17). OK.

This last sum is ($-6 + 17 = 11 = -b$). The 2 real roots are (-6) and (17).

Consequently, there is no further need to do factoring by grouping and to solve the 2 binomials for x .

Note. If we compose the factors of $c = -102$ differently, the results will be the same. Proceeding: (1, -102), (2, -51), (3, -34), (6, -17). This last sum is $(6 - 17 = -11 = b)$. The opposite sum $(-6, 17) = -b$ gives the 2 real roots: -6 and 17.

TIP 2. When both roots have same sign ($ac > 0$), compose factors of c with all positive numbers.

Example 4. Solve: $x^2 - 28x + 96 = 0$.

Solution. Both real roots are positive ($ac > 0$). Compose factors of $c = 96$ with all positive numbers. Proceeding: (1, 96), (2, 48), (3, 32), (4, 24). This last sum is: $4 + 24 = 28 = -b$. Then, the 2 real roots are 4 and 28.

Example 5. Solve: $x^2 + 39x + 108 = 0$.

Solution. Both real roots are negative (Rule of Signs). Compose factors of $c = 108$ with all positive numbers. Proceeding: (2, 54), (3, 36). OK. The last sum is $3 + 36 = 39 = b$. The opposite sum $(-b)$ give the 2 real roots (-3) and (-36) .

CASE 2. SOLVING QUADRATIC EQUATIONS, TYPE $ax^2 + bx + c = 0$ ($a \neq 1$)

This Transforming Method method proceeds through **3** steps:

Step 1. Transform the given quadratic equation in standard form $ax^2 + bx + c = 0$ **(1)** into a simplified equation, with $a = 1$, and with a new constant ($a*c$).

The transformed equation has the form: $x^2 + bx + a*c = 0$ **(2)**.

Solve $y = 8x^2 - 22x - 13 = 0$

Transformed equation: $y' = x^2 - 22x - 104 = 0$ ($a*c = 8*-13 = -104$)

Step 2. Solve this transformed equation **(2)** by the method explained in **CASE 1** that immediately obtains the 2 real roots: **X1** and **X2**. The method proceeds by composing factor pairs of (ac) , then, by finding the pair whose sum equal to $(-b)$.

Step 3. Divide both **X1** and **X2** by the coefficient **a** to get the 2 real roots **x1** and **x2** of the original equation **(1)**: **x1** = $X1/a$, and **x2** = $X2/a$.

EXAMPLES OF SOLVING BY THE NEW “TRANSFORMING METHOD”

Example. Solve: $8x^2 - 22x - 13 = 0$. **(1)**

Solution. Solve the transformed equation: $x^2 - 22x - 104 = 0$ (2). The 2 real roots have opposite signs ($ac < 0$). Compose factors of $a*c = -104$ with all first numbers being negative.

Proceeding: $(-2, 52)(-4, 26)$. This last sum is $26 - 4 = 22 = -b$. The 2 real roots of the transformed equation (2) are: $X1 = -4$, and $X2 = 26$.

Back to the original equation (1), the 2 real roots are: $x1 = X1/a = -4/8 = -1/2$, and $x2 = X2/a = 26/8 = 13/4$.

Example 7. Solve: $16x^2 - 62x + 21 = 0$ (1)

Solution. Solve the transformed equation: $x^2 - 62x + 336 = 0$ (2). Both real roots are positive. Compose factors of $a*c = 336$ with all positive numbers.

Proceeding: $(2, 168)(4, 82)(6, 56)$. This last sum is $56 + 6 = 62 = -b$. Then, the 2 real roots of (2) are: $X1 = 56$, and $X2 = 6$.

Back to original equation (1), the 2 real roots are: $x1 = 56/16 = 7/2$, and $x2 = 6/16 = 3/8$.

Example 8. Solve: $18x^2 + 267x - 91 = 0$. (1).

Solution. Solve the transformed equation: $x^2 + 267x - 1638 = 0$. (2).

Roots have opposite signs. Compose factor pairs of $a*c = -1638$ with all negative first numbers. Use a calculator to work faster with factor pairs.

Proceeding: $(-2, 819)(-3, 546)(-6, 273)$. This last sum is $(273 - 6 = 267) = b$. The opposite pair $(6, -273) = -b$, gives the 2 real roots of equation (2):

$X1 = 6$, and $X2 = -273$. Therefore, the 2 real roots of the original equation (1) are: $x1 = X1/a = 6/18 = 1/3$, and $x2 = X2/a = -273/18 = -91/6$.

CONCLUSION

The main strong points of the new “Transforming Method” are: fast, systematic, no guessing, no lengthy factoring by grouping, and no solving binomials. It is very effective to solve all types of quadratic equations that can be factored (when a , b , c , and ac are large numbers).

References:

- The transformation of an equation in standard form $ax^2 + bx + c = 0$ into an equation in simplified form $x^2 + bx + a*c = 0$ was publicly presented in 3 math articles (Google, Yahoo, Bing Search):
 1. AC Method for factoring- Regent Exam Prep Center at www.regentsprep.org
 2. “A different way to solve Quadratic – The Bluma Method”.
 3. Factoring trinomials – Simplified AC Method at: 2000clicks.com/mathhelp

