SOLVING QUADRATIC EQUATIONS BY THE NEW "TRANSFORMING METHOD"

(By Nghi H Nguyen)

This new method is fast, convenient, systematic, with no guessing, and it is applicable whenever the quadratic equation can be factored. It can immediately obtain the 2 real roots without factoring by grouping, and solving the binomials.
It uses in its solving process three features:

- The Rule of Signs For Real Roots of a quadratic equation that shows the signs (- or +) of the 2 real roots to select a better solving approach.
- The Diagonal Sum Method for solving quadratic equations type $x^2 + bx + c = 0$, $(a = 1)$.
- The transformation of a given quadratic equation in standard form $ax^2 + bx + c = 0$ (1) into another equation (2) in the form $x^2 + bx + c = 0$, $(a = 1)$ for a better solving approach. (See References)

RECALL THE RULE OF SIGNS FOR REAL ROOTS OF A QUADRATIC EQUATION

- If **a** and **c** have opposite signs, the 2 real roots have opposite signs.

Example. The equation $x^2 - 8x – 9 = 0$ has 2 real roots with different signs: -1 and 9.

- If **a** and **c** have same sign, the 2 real roots have same sign.
  - If **a** and **b** have same sign, both real roots are negative

Example: The equation $x^2 + 17x + 15 = 0$ has 2 real roots both negative: -1 and -15.

  - If **a** and **b** have different signs, both real roots are positive.

Example: The equation $5x^2 – 14x + 9 = 0$ has 2 real roots both positive: 1 and 9/5.

SOLVING QUADRATIC EQUATIONS BY THE DIAGONAL SUM METHOD

When a = 1 – Solving equation type $x^2 + bx + c = 0$.

In this case, solving results in finding 2 numbers knowing their sum (-b) and their product (c). Applying the Rule of Signs, the solving process using the Diagonal Sum Method (Google or Yahoo Search) is simple, fast, and **doesn't require factoring by grouping and solving the 2 binomials.** It proceeds following these 2 Tips.

a. TIP 1. When roots have different signs (**a** and **c** different signs). Compose the factor pairs of **c** with all first numbers of the pairs being negative. Stop composing when you find the pair whose sum is equal to **(b)**, or **(-b).** If you don't find it, then the equation can't be factored, and you should use the quadratic formula to solve it.

Example 1. Solve: $x^2 – 11x – 102 = 0$.

Solution. The Rule of Signs indicates roots have different signs. First, compose factor pairs of c = -102 with all first numbers being negative. Proceeding: (-1, 102),(-2, 51),(-3, 34),(-6, 17). OK.. This last sum is 17 – 6 = 11 = -b. Consequently, the 2 real roots are -6 and 17. There is no need to factor by grouping and to solve the 2 binomials for x.

Note. If we compose the factors of c = -102 differently, the results will be the same. Proceeding: (1, -102),(2, -51),(3, -34),(6, -17). This last sum is 6 – 17 = -11 = b. According to the Diagonal Sum Rule, when the diagonal sum equals to (b) the answers are the opposite. Then, the 2 real roots are -6 and 17.

b. **TIP 2. When roots have same sign** (**a** and **c** same sign), compose factors of **c** with:

- all positive numbers if both roots are positive
- all negative numbers if both roots are negative

Example 2. Solve: x^2 – 28x + 96 = 0.

Solution. Both real roots are positive. Compose factors of c = 96 with all numbers being positive. Proceeding: (1, 96),(2, 48),(3, 32),(4, 24). OK. This last sum is 4 + 24 = 28 = -b. Then, the 2 real roots are 4 and 28. No needs for factoring and solving binomials!

Note. If we compose the factors of c = 96 differently, the outcome is still the same. Proceeding: (-1, -96),(-2,-48),(-3, -32),(-4, -24). This last sum is (-4 – 24) = -28 = b. The real roots make the negative of this pair. Then, the real roots are 4 and 24.

Example 3. Solve: x^2 + 39x + 108 = 0.

Solution. Both real roots are negative. Compose factors of c = 108 with all numbers negative. Proceeding: (-1, -108),(-2, -54),(-3, -36). OK. The last sum is -3 - 36 = -39 = -b. Then, the 2 real roots are -3 and -36. No need for factoring and solving binomials!

SOLVING QUADRATIC EQUATIONS BY THE NEW "TRANSFORMING METHOD"

This new method proceeds through **3** steps:

Step 1. Transform the given quadratic equation **(1)** in standard form ax^2 + bx + c = 0 into a new equation **(2)** with a = 1 and with the new constant C = a*c. The new equation has the form: x^2 + bx + a*c = 0 **(2).**

Step 2. Solve this new equation **(2)** by the **Diagonal Sum Method** that immediately obtains the 2 real roots without factoring by grouping or solving binomials. Suppose the 2 found real roots of the equation **(2)** are: **y1** and **y2.**

Step 3. Divide both **y1** and **y2** by **a** to get the 2 real roots **x1** and **x2** of the original equation **(1).**

Important Notes.

1. When composing factor pairs of a*c, always start from the first factor (-1, c), or (1, c), or (-1, -c), then gradually go up the chain. Stop composing when you find the factor pair

whose sum is equal to **(b)**, or **(-b).** If you can't find this pair, then the given quadratic equation can't be factored, and consequently you must probably use the quadratic formula to solve it.

2. To save time when composing the long chain of factor pairs of **a*c**, in case it is a very large number, consider the relative values of **b** compared to **a*c**.

a. If **b** is relatively as large as a***c**, start the factor chain from the beginning.
Example 4: Solve: 5x^2 - 18x - 8 = 0. Compose factor pairs of a*c = -40 from the beginning. Proceeding: (-1, 40),(-2, 20). OK.

b. If **b** is very small as compared to **a***c**, start the factor chain from the middle.
Example 5. Solve: 15x^2 – 53x + 16 = 0. Since b = 5 is small number as compared to a*c = 15*16 = 240, start composing factor pairs from the middle of the chain. Proceeding:…..(3, 80),(4, 60),(5, 48). OK.

EXAMPLES OF SOLVING BY THE NEW "TRANSFORMING METHOD"

Example 6. Solve: 8x^2 – 22x – 13 = 0. **(1)**

Solution. Solve the transformed equation: x^2 – 22x – 104 = 0 **(2),** by the Diagonal Sum Method. Roots have different signs. Compose factors of a*c = -104 with all first numbers being negative. Proceeding: (-1, 104)(-2, 52)(-4, 26). This last sum is 26 – 4 = 22 = -b. The 2 real roots of the transformed equation (2) are: **y1** = -4, and **y2** = 26.
Consequently, the 2 real roots of (1) are: **x1** = -4/8 = -1/2, and **x2** = 26/8 = 13/4.

Example 7. Solve: 16x^2 – 62x + 21 = 0 (1)

Solution. Solve the transformed equation: x^2 – 62x + 336 = 0 (2). Both real roots are positive. Compose factors of a*c = 336 with all numbers being positive. Proceeding: (1, 336) (2, 168)(4, 82)(6, 56). This last sum is 56 + 6 = 62 = -b. Then, the 2 real roots are: **y1** = 56, and **y2** = 6. Back to the original equation (1), the 2 real roots are: **x1** = 56/16 = 7/2, and **x2** = 6/ 16 = 3/8.

Example 8. Solve: 12x^2 + 29x + 15 = 0. (1).

Solution. Solve the transformed equation: x^2 + 29x + 180. (2), by the Diagonal Sum Method Both roots are negative. Compose factor pairs of a*c = 180 with all negative numbers. Proceeding: (-1, -180)(-2, -90)(-3, -60)(-4, -45)(-5, -36)(-6, -30)(-9, -20). This last sum is (-29) = -b. The 2 real roots of (2) are: y1 = -9 and y2 = -20. Then, the 2 real roots of the original equation (1) are: **x1** = -9/12 = -3/4, and **x2** = -20/12 = -5/3.

CONCLUSION

The main strong points of the new "Transforming Method" are: simple, fast, systematic, no guessing, no more factoring by grouping, and no more solving binomials.

References:

- The transformation of an equation in standard form into an equation in the form x^2 + bx + c = 0 was publicly presented in 2 math articles (Google or Yahoo Search):

1. AC Method for factoring- Regent Exam Prep Center at www.regentsprep.org
2. "A different way to solve Quadratic – The Bluma Method".
3. Factoring trinomials – Simplified AC Method at: 2000clicks.com/mathhelp

- The Diagonal Sum Method was publicly presented in math articles titled:"Solving quadratic equations by the Diagonal Sum Method" (Google or Yahoo Search).

(This article titled "Solving quadratic equations by the new "Transforming Method" was written by Nghi H Nguyen, co-author of the new Diagonal Sum Method for solving quadratic equations)