SHORTCUT IN SOLVING LINEAR EQUATIONS
(Basic Step to Improve math skills of high school students)

(by Nghi H. Nguyen)

Most of the immigrant students who first began learning Algebra I in US high schools found the existing linear equation solving process very different from what they had learned in their countries.
Let's see what the differences are and how can the existing solving process be improved?
The basic difference shows itself in the way we teach students on how to solve a linear equation. We teach them doing it in 2 steps: simple step for simple equation and multiples steps for more complicated equations.
In contrary, the world wide solving process, a process called **Shortcut** by the author of this article, performs by moving **(transposing)** terms,
The two different processes are generally explained below then they are compared each other through selected examples.

SOLVING LINEAR EQUATIONS IN SIMPLE STEP.

<u>Example 1</u>. Solve   x – 5 = 2
Solution.   **+5   +5**
                              x = 7
We teach students to add +5 by writing +5 in **both sides** of the equation, one line down from the equation line.
Some math books write **+5** on both sides of the equation, on the **same** equation line
                  x – 5 **+ 5** = 2 **+ 5**
                          x = 7
<u>Example 2</u>. Solve: 3x = 9
Solution.   3x = 9
                      :3   :3
                        x = 3
Students are taught to write **(:3)** on both sides of the equations, one line down from the equation line. Some books tell students to multiply by (**1/3**) on both sides.
                    3x = 9
              3x .(**1/3**) = 9.(**1/3**)
                  x = 3

<u>Example 3.</u> Solve   5x/11 = 3
Solution. Students are taught to divide the 2 sides by the fraction (**5/11**), meaning to multiply by the inverse of the fraction (5/11) that is the fraction (**11/5**).
                  5x/11.(**11/5**) = 3.(**11/5**)
                          x = 33/5

SOLVING LINEAR EQUATIONS IN MULTIPLES STEPS.

<u>Example 4</u>. Solve: 5x + a – 2b - 5 = 2x - 2a + b -3
Solution. We teach students to solve this equation in multiple steps.

Step 1. Simplify the quantities containing x.

$$5x + a - 2b - 5 = 2x - 2a + b - 3$$
$$\textbf{-2x} \qquad \textbf{-2x}$$
$$3x + a - 2b - 5 = -2a + b - 3$$

Step 2. Simplify the quantities containing letters and numbers. Some math books divide this into 2 small steps, simplify the letters first, then the numbers second.

$$3x + a - 2b - 5 = -2a + b - 3$$
$$\textbf{-a + 2b + 5} \quad \textbf{-a + 2b + 5}$$
$$3x = -3a + 3b + 2$$

Step 3. Make the variable x alone in one side of the equation

$$3x = -3a + 3b + 2$$
$$\textbf{:3} \qquad \textbf{:3}$$
$$x = -a + b + 2/3$$

<u>Example 5</u>. Solve   5x/3 – 7 = 3x/2 – 3

Solution.   **-3x/2   -3x/2**
$$5x/3 - 3x/2 - 7 = -3$$
$$x/6 - 7 = -3$$
$$\textbf{+7} \qquad \textbf{+7}$$
$$x/6 = 4$$
$$\textbf{:1/6} \quad \textbf{:1/6}$$
$$x = 4.(6) = 24$$

Remark. There are **double writings** of terms (variables, letters, numbers, constants), on both sides of the equation, in every step of the solving process. This double writing of terms looks simple and easy at the beginning of Algebra I level, will become confusing and abundant when later students deal with long/complicated terms in higher levels of equations and inequalities. In addition, this double writing takes too much time and easily leads to errors and mistakes.

I strongly propose that the shortcut in solving linear equations be introduced and taught at the beginning of Algebra I level.

Basic concept of the shortcut in solving linear equations

"When a term moves to the other side of the equation, its operation changes to the inverse operation".

The inverse operation of an addition is a subtraction. Multiplication and division are inverse operations.

Operational Rule of the Shortcut.

When a term (n) moves to the other side of an equation:

a. An addition (+n) changes to a subtraction (-n)
   Example: $x + n = 3$

   $x = 3 - n$ (+n moves to right side and becomes –n)

b. A subtraction (-n) changes to an addition (+n).
   Example: $x - n = 5$

   $x = 5 + n$ (-n moves to right side and becomes +n)

c. A multiplication (.n) changes to a division (1/n).

   Example $x.(n) = 4$

   $x = 4/n$ (n moves to right side and becomes 1/n)

d. A division (1/n) changes to a multiplication.
   Example: $x/n = 7$

   $x = 7.n$

Examples of solving linear equations using Shortcut.

Example 1. Solve: $x - 3 + 7 = 8$
Solution. $x = 8 + 3 - 7 = 4$ (-3 and +7 move to right side)

Example 2. Solve: $x - m + 3 = 2m + 1$

Solution. $x = 2m + 1 + m - 3 = 3m - 2$ (-m and +3 move to right side)

Example 3. Solve: $2x - a + 4 = x + 3a - 1$

Solution. $2x - x = 3a - 1 + a - 4$ (-a + 4 move to right side)
                                  $x = 4a - 5$

Example 4. Solve: $5x - 7 = 2x + 5$

Solution. $5x - 2x = 5 + 7$ (-7 moves to right side and 2x moves to left side)
                              $3x = 12$
                                $x = 12/3 = 4$ (3 moves to right side)

Example 5. Solve: $ax/b = c/d$

Solution. $x = bc/ad$ (a / b moves to right side and becomes b/a)

Example 6. Solve: $3/5 = 4/x - 2$

Solution. $x - 2 = 5(4)/3 = 20/3$ (Move (x - 2) to left side, and 3/5 to right side)
                              $x = 2 + 20/3 = 26/3$ (Move -2 to right side)

Advantages of the Shortcut

1. The shortcut proceeds solving faster since it helps avoid the double writing of terms (variables, letters, numbers, constants) on both sides of the equation in every solving step.

Example 7. Solve: $5x - m + 4 = 2m + 2x + 9$

Solution.

Multiple steps without shortcut   With Shortcut
$5x - m + 4 = 2m + 2x + 9$ $5x - m + 4 = 2m + 2x + 9$
    -2x   -2x $5x - 2x = 2m + 9 + m - 4$
$3x - m + 4 = 2m + 9$ $2x = 3m + 5$
   +m - 4   +m - 4 $x = m + 5/3$
        $3x = 3m + 5$
      :3   :3
        $x = m + 5/3$

Example 8. Solve: $(t-1)/(t + 1) = 3/(x - 2)$

Solution.

Multiple steps without Shortcut   With Shortcut

$(t - 1)/(t + 1) = 3/(x - 2)$ $(t - 1)/(t + 1) = 3/(x - 2)$
$(t - 1)(x - 2) = 3(t + 1)$ $x - 2 = 3(t + 1)/(t - 1)$
$(t - 1)x - 2(t - 1) = 3(t + 1)$ $x = 2 + 3(t + 1)/(t - 1)$
     +2(t − 1)   **+2(t − 1)**
$(t - 1)x = 3(t + 1) + 2(t - 1)$
 :(t − 1)   **:(t − 1)**
     $x = [3(t + 1) + 2(t - 1)]/(t - 1)$

2. The double writing of terms looks simple and easy to understand at the beginning of Algebra I level. However, when the terms become long/complicated, this abundant double writing takes too much time and is usually the main causes for errors/mistakes. Students may feel inconvenient when years later using the double writing of terms to deal with complex equations and inequalities, such as equations in third degree and higher, rational equations, parametric equations, trig equations and inequalities…

3. The Shortcut provides students with a "good habit" to check and follow the moving terms in both sides of the equation using a simple rule: "no **new** terms added, no **missing** terms after every **move**". Therefore, students can avoid committing errors and mistakes. This "good habit" will help them later to deal with more complex equations and inequalities in future math study.

Example 9. Solve: $2(x - m + 1)/(x - 2 - m) = 5/3$

Solution. $6(x - m + 1) = 5(x - 2 - m)$
                     $6x - 6m + 6 = 5x - 10 - 5m$ (6 terms)
                     $6x - 5x = -10 - 5m + 6m - 6$ (6 terms)
                          $x = m - 16$

Example 10. Solve: $(t - 1)/4x = (2t - 3)/(x + 2)$
Solution.

Multiple steps without Shortcut   With Shortcut

$(t - 1)(x + 2) = 4x(2t - 3)$ $(t - 1)(x + 2) = 4x(2t - 3)$
$(t - 1)x + 2(t - 1) = 8tx - 12x$ $tx - x + 2(t - 1) = 8tx - 12x$ (5 terms)
 $tx - x + 2(t - 1) = 8tx - 12x$ $tx - x - 8tx + 12x = -2(t - 1)$ (5 terms)
   -8tx + 12x   **-8tx + 12x** $x(11 - 7t) = -2(t - 1)$
$-7tx + 11x + 2(t - 1) = 0$ $x = -2(t - 1)/(11 - 7t)$
          - 2(t − 1)   **- 2(t − 1)**

$$x(11 - 7t) = -2(t - 1)$$
:(11 – 7t)   : (11 – 7t)
$$x = -2(t - 1)/(11 - 7t)$$

4   Using "smart moves"

The smart move of the shortcut allows students to logically avoid doing operations such as cross multiplications and distributive multiplications that are sometimes unnecessary.

<u>Example 11.</u> Solve: $(x + 5) /3 = 7/ 4$

Solution. Don't automatically proceed cross multiplication and distributive multiplication.

$(x - 5) = 7(3)/4 = 21/4$ (Leave x – 5 on place; move 3 to right side)
$x = 5 + 21/4 = 41/4$ (Move -5 to right side)

<u>Example 12.</u> Solve: $3t/ (t - 1) = 5/(x - 7)$

Solution. Don't proceed cross multiplication and distributive multiplication.

$(x - 7) = 5(t - 1)/3t$ (Move x – 7 to left side; move 3t/(t – 1) to right side)
$x = 7 + 5(t - 1)/3t$ (Move -7 to right side)

<u>Example 13</u>. Solve: $a/b = c(x - 2)/d$

Solution. Don't proceed cross multiplication and distributive multiplication.
$(x - 2) = bc/ad$ (keep x - 2 on place, move a/b to right side; switch sides)
$x = 2 + bc/ad$

5   The shortcut easily helps to transform math and science formulas.

<u>Example 14.</u> Transform the formula $V2 = V1R2/(R1 + R2)$ to get R2 in terms of other letters.

Solution.
<u>Multiple steps without Shortcut</u>   <u>With Shortcut</u>
  $V2(R1 + R2) = V1R2$ (Cross multiplication) $V2(R1 + R2) = V1R2$
$V2R1 + V2R2 = V1R2$ $V2R1 + V2R2 = V1R2$ (3 terms)
     -V1R2   -V1R2 $V2R2 - V1R2 = -V2R1$ (3 terms)
$V2R1 + V2R2 - V1R2 = 0$ $R2(V2 - V1) = -V2R1$
             -V2R1   -V2R1 $R2 = -V2R1/(V2 - V1)$
$R2(V2 - V1) = -V2R1$
: (V2 - V1)   : (V2 - V1)
        $R2 = -V2R1/(V2 - V1)$

Example 15. Transform the formula $1/f = 1/d1 + 1/d2$ to get d2 in terms of other letters.

Solution.

Multiple steps without shorcut	With Shortcut
$1/f = 1/d1 + 1/d2 = (d2 + d1)/d1d2$	$1/f = 1/d1 + 1/d2 = (d2 + d1)/d1d2$
$f(d1 + d2) = d1d2$	$1/d2 = 1/f - 1/d1 = (d1 - f)/fd1$
$fd1 + fd2 = d1d2$	$d2 = fd1/(d1 - f)$ (Inverse the equation)
-d1d2 **-d1d2**	
$fd1 + fd2 - d1d2 = 0$	
-fd1 **-fd1**	
$d2(f - d1) = -fd1$	
: (f - d1) : (f - d1)	
$d2 = -fd1/(f - d1)$	

CONCLUSION.

I have seen many high school students struggling when solving various algebraic equations and inequalities by using the **double writing** of terms on both sides of the equation.

I strongly suggest that the Shortcut in solving linear equation be introduced and taught in high schools. The shortcut is actually a very popular worldwide equation solving process. Students in the whole world are learning and performing the shortcut, why do American students do it differently?

Introducing the shortcut at the beginning of Algebra 1 may be the first basic step to improve math skills of American students.

(This article was written by Nghi H. Nguyen, co-author of the new
Diagonal Sum Method for solving quadratic equations)