THE QUADRATIC FUNCTION AND THE NEW QUADRATIC FORMULA IN INTERCEPT FORM

(By Nghi H Nguyen – Updated on Nov 18, 2014)

There are so far 8 common methods to solve a quadratic equation in standard form: $ax^2 + bx + c = 0$. They are: quadratic formula, completing the square, factoring FOIL method, graphing, the Bluma Method, the Diagonal Sum Method, the factoring AC method, and the new "Transforming Method", recently introduced on Google and Yahoo Search.

If the given quadratic equation can be factored, the new **Transforming Method** is the best solving method since it can immediately obtain the 2 real roots without factoring by grouping and solving binomials. In case the given quadratic equation can't be factored, the **quadratic formula** would be the obvious choice to solve it. There is a new and improved quadratic formula, called: "**The new quadratic formula in intercept form**", that is simpler and easier to remember than the classical formula.

The quadratic function in intercept form

The graph of the quadratic function in standard form $f(x) = ax^2 + bx + c$ is a **parabola** that may intercept the x-axis at 2 points, unique point, or no point at all. This means a quadratic equation $f(x) = ax^2 + bx + c = 0$ may have 2 real roots, one double real root, or no real roots at all (complex roots).

We can write f(x) in the form: $y = a(x^2 + b/a\, x + c/a)$ **(1)**.

Recall the development of the quadratic formula:

$x^2 + bx/a + (b^2/4a^2 - b^2/4a^2) + c/a = 0$

$(x + b/2a)^2 - (b^2 - 4ac)/4a^2 = 0$

$(x + b/2a)^2 - d^2/4a^2 = 0.$ (Call $\mathbf{d}^2 = b^2 - 4ac$)

$(x + b/2a + \mathbf{d}/2a)(x + b/2a - \mathbf{d}/2a) = 0$ **(2)**

Replace this expression (2) into the equation (1), we get the quadratic function f(x) written in **intercept form**:

$f(x) = a(x - x1)(x - x2)$ (x1 and x2 are the 2 x-intercepts, or roots of f(x) = 0)

$f(x) = a(x + b/2a + d/2a)(x + b/2a - d/2a)$ (3)

The quadratic formula in intercept form. From the equation (3), we deduct the formula:

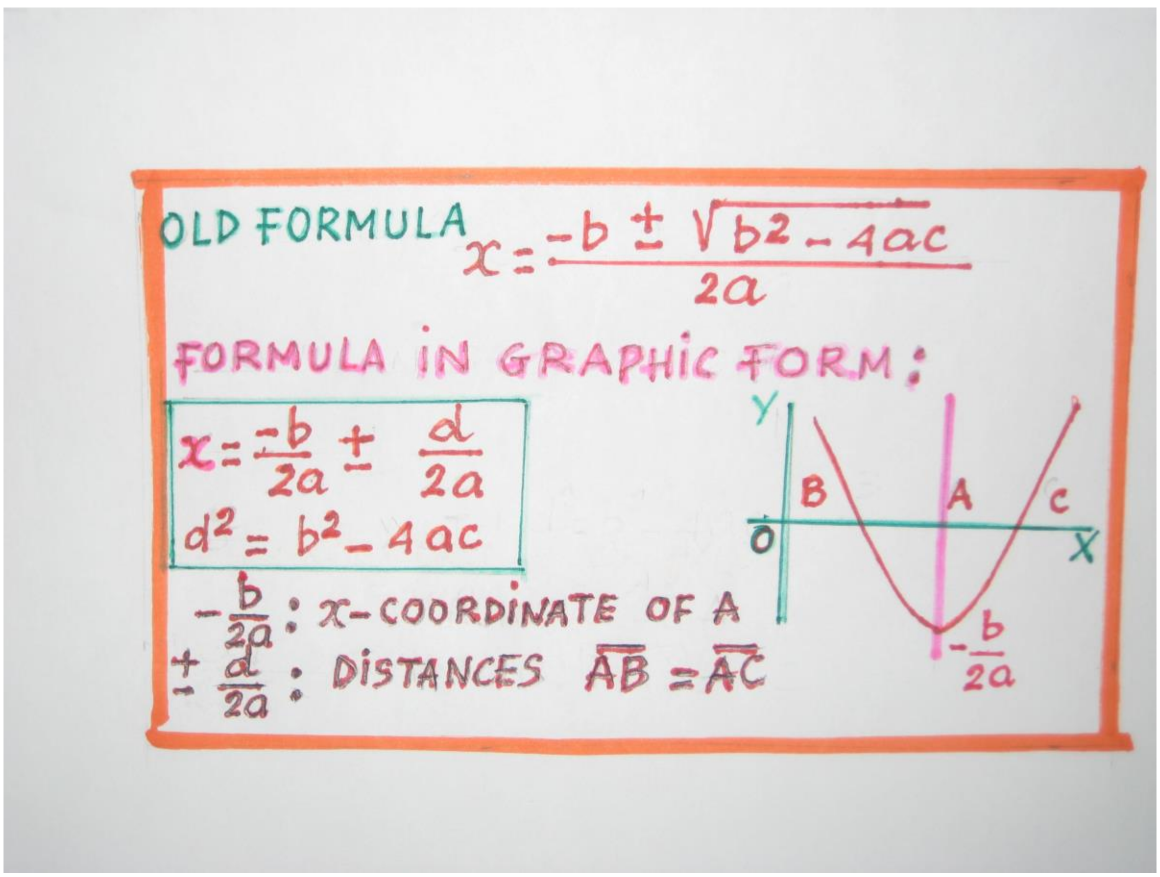
$$x = -b/2a \pm d/2a \quad (4)$$

In this formula, x being the 2 roots of the quadratic equation f(x) = 0:

- The quantity **(–b/2a**) represents the x-coordinate of the parabola axis.
- The 2 quantities **(d/2a)** and **(-d/2a)** represent the 2 distances AB and AC from the parabola axis to the two x-intercepts (real roots) of the parabola.
- The quantity **d** can be zero, a number, or imaginary.
- If d = 0: there is a double root at x = -b/2a.
- If d is a number (real or radical), there are two x-intercepts, meaning two real roots.
- If d is imaginary, there are no real roots. The parabola doesn't intercept the x-axis.

- The unknown quantity **d** can be computed from the constants a, b, c by this relation (5):

$$d^2 = b^2 - 4ac \quad (5)$$



We can easily obtain this relation by writing that the product (x1.x2) of the 2 real roots is equal to (c/a).

$$(-b/2a + d/2a)(-b/2a - d/2a) = c/a$$

$$b^2 - d^2 = 4ac \rightarrow d^2 = b^2 - 4ac$$

If $d^2 = 0$, there is double root at $x = -b/2a$

If $d^2 > 0$, there are 2 real roots.

If $d^2 < 0$, there are no real roots, there are complex roots

To solve a quadratic equation, first find the quantity d by the relation (5), then find the 2 real roots by the formula (4).

REMARK. This new quadratic formula in intercept (graphic) form is simpler and easier to remember than the classic formula since students can relate it to the x-intercepts of the parabola graph. In addition, the two quantities **(d/2a)** and **(-d/2a)** make more sense about distance than the classical quantity: √(b^2 – 4ac).

Examples of solving quadratic equations by the quadratic formula in intercept (graphic) form.

Example 1. Solve: $4x^2 - 12x + 9 = 0$.

Solution. First, find d by the relation (5):
$d^2 = b^2 - 4ac = 144 - 144 = 0 \rightarrow d = 0$
The equation has double root at $x = -b/2a = 12/8 = 3/2$.

Example 2. Solve: $2x^2 - 3x + 7 = 0$

Solution. First find d^2 by the relation (5).
$d^2 = 9 - 56 = -47$ (< 0)
d is imaginary, there are no real roots. There are 2 complex roots.

Example 3. Solve: $3x^2 + 16x - 12 = 0$.

Solution. $d^2 = 256 + 144 = 400 = 20^2$
Next, find the 2 real roots by the formula (4).
x1 = -16/6 + 20/6 = 4/6 = 2/3
x2 = -16/6 – 20/6 = -36/6 = -6.

Example 4. Solve: $7x^2 + 18x - 25 = 0$.

Solution. $d^2 = 324 + 700 = 1024 = 32^2$
$x1 = -18/14 + 32/14 = 14/14 = 1$
$x2 = -18/14 - 32/14 = -50/14 = -25/7$

Example 5. Solve: $2x^2 + 12x + 17 = 0$

Solution. $d^2 = 144 - 136 = 8 \rightarrow d = 2.83$
$x1 = -12/4 + 2.83/4 = -9.17/4 = -2.29$
$x2 = -12/4 - 2.83/4 = -14.83/4 = -3.70$

Example 6. Solve: $5x^2 - 10x - 3 = 0$.

Solution. $d^2 = 100 + 60 = 160 \rightarrow d = 12.65$
$x1 = 10/10 + 12.65/10 = 22.65/10 = 2.26$
$x2 = 10/10 - 12.65/10 = -2.65 /10 = -0.26$

NOTE. When the given quadratic equation can be factored, its 2 real roots are usually in the form of two fractions. The quantity d^2 should be a perfect square and **d** should be a **whole number**. So, students are advised to proceed solving by the formula in intercept form, mentally or by using calculators, through 2 steps. First step, find d by the relation **(5).** Second step, **algebraically** calculate the 2 real roots by the Formula **(4).** In case **d** is a whole number, make sure that the 2 real roots (answers) be in the form of 2 fractions and not in decimals as given by calculators.

[This article was written by Nghi H. Nguyen, author of the new Transforming Method (Mathematics Magazine Search) for solving quadratic equations – Updated on Nov 18, 2014]