SOLVING QUADRATIC EQUATIONS BY THE NEW "AC METHOD"

(by Nghi H. Nguyen – Updated Jan 27, 2015**)**

So far, the **factoring** "AC Method" (YouTube) has been the most popular factoring method to solve a quadratic equation in standard form, $ax^2 + bx + c = 0$, that can be factored. Another factoring method, called the "Box Method" (YouTube) offers a similar solving approach.

This article generally explains this **factoring** method, then, it suggests a new and improved solving method, called "The New AC Method".

THE FACTORING "AC METHOD".

Given a quadratic equation in standard form $ax^2 + bx + c = 0$, that can be factored, the AC method proceeds to factor this equation into 2 binomials in x by replacing in the equation the term (bx) by the 2 terms (b1x) and (b2x) that satisfy these 2 conditions:

1. The product b1*b2 = a*c
2. The sum (b1 + b2) = b.

Example 1. Solve: $x^2 - 11x - 102 = 0$.

Solution. Find 2 numbers that the product is (b1*b2) = c = -102 and the sum (b1 + b2) = b = -11. Proceed by composing all factors of ac = -102. Proceeding: (-1, 102),(1, -102),(-2, 51),(2, -51)(-3, 34)(3, -34),(-6, 17),(6, -17).OK. The last sum is: (6 - 17) = -11 = b. Then b1 = 6 and b2 = -17. Next, replace in the equation the term (-11x) by the 2 terms (6x) and (-17x), then factor by grouping:
$x^2 - 11x - 102 = x^2 + 6x - 17x - 102 = x(x + 6) - 17(x + 6) = (x + 6)(x - 17) = 0$.
Finally, solve the 2 binomials to get the 2 real roots.
$(x + 6) = 0 \rightarrow x = -6$
$(x - 17) = 0 \rightarrow x = 17$.

Important Remark 1. The 2 found terms (b1= 6) and (b2 = -1) are, in fact, the opposite values of the 2 real roots. Consequently, **it is unnecessary to do the next lengthy steps** (factoring by grouping and solving the 2 binomials). Change the second condition to (b1 + b2) = -b, then we get the right 2 real roots.

Example 2. Solve: $15x^2 - 53x + 16 = 0$ (1) (AC = 15*16 = 240)

Solution. Find 2 numbers that their product is a*c = 240 and their sum is b = -53.
Proceeding: (1, 240),(2, 120),(3, 80),(4, 60)(5, 48)
(-1, -240),(-2, -120),(-3, -80)(-4, -60),(-5, -48). OK. → b1 = -5 and b2 = -48

Next, replace in the equation the term (-53x) by the 2 terms (-5x) and (-48x), then factor by grouping:
$15x^2 - 5x - 48x + 16 = 5x(3x - 1) - 16(3x -1) = (3x - 1)(5x - 16) = 0$.

Next, solve the 2 binomials:
$(3x – 1) = 0 \rightarrow x = 1/3$
$(5x – 16) = 0 \rightarrow x = 16/5$

Important Remark 2. The 2 numbers b1 = -5 and b2 = -48 are in fact the opposite values of the 2 real roots of a simplified equation: $x^2 - 53x + 240 = 0$ (2). Solve this simplified equation (2) to get (b1 = 5) and (b2 = 48), then divide them by (a = 16), we can get the 2 real roots of the original equation (1): x1 = b1/15 = 5/15 = 1/3; and x2 = b2/15 = 48/15 = 16/5.
It is unnecessary to do the next lengthy steps: factor by grouping and solving the binomials.

Remark 3. The existing factoring AC Method can be considerably improved if we apply the Rule of Sign for real roots into its solving approach.

RECALL THE RULE OF SIGN FOR REAL ROOTS.

a. When **a** and **c** have different signs, both real roots have different signs.

Example. The equation $5x^2 + 8x - 13 = 0$ has 2 real roots with opposite signs: (1) and (-13/5)
Example. The equation $7x^2 – 8x – 15 = 0$ has 2 real roots with opposite signs: (-1) and (15/7)

b. When **a** and **c** have the same sign, both real roots have the same sign.

- If **a** and **b** have opposite signs, both real roots are positive.

Example: The equation $8x^2 - 11x + 3 = 0$ has 2 real roots both positive: 1 and 3/8.
Example: The equation $13x^2 – 21x + 8 = 0$ has 2 real roots both positive: 1 and 8/13

- If **a** and **b** have same sign, both real roots are negative.

Example: The equation $8xv + 11x + 3 = 0$ has 2 negative real roots: (-1) and (-3/8).
Example: The equation $13x^2 + 23x + 10 = 0$ has 2 negative real roots: (-1) and (-10/13).

THE NEW "AC METHOD".

This method proceeds to find 2 numbers b1 and b2 that satisfy these 3 conditions:

1. The product b1*b2 = ac
2. The sum (b1 + b2) = -b (instead of b)
3. Application of the **Rule of Signs** into the solving process.

CASE 1. When a = 1. Solving quadratic equation type $x^2 + bx + c = 0$

When **a = 1**, solving results in finding 2 numbers knowing their product (c) and their sum (-b). This method can **immediately obtain the 2 real roots** of the quadratic equation by composing

factor pairs of **c**. The factor pair, whose sum equals to **(b),** or **(-b),** gives the 2 real roots. **There is no need for factoring by grouping or solving the 2 binomials.**
This method composes factor pairs of **c** following these 3 Tips.

TIP 1. When **a** and **c** have opposite signs, roots have opposite signs. Compose factor pairs of **c** with all first number being negative.

<u>Example 1</u>. Solve: $x^2 - 11x - 102 = 0$.

Solution. Since **a** and **c** have opposite signs, roots have opposite signs. Compose the factor pairs of (c = -102) with all first numbers being negative.
Proceeding: (-1, 102),(-2, 51),(-3, 34),(-6, 17). OK!.
The sum of the last pair is: 17 – 6 = 11 = -b. Then the 2 real roots are -6 and 17. There is no need for factoring by grouping and solving the 2 binomials for x.

TIP 2. When both real roots are positive (a and c same sign, a and b different signs), compose factor pairs of **c** with all positive numbers.

<u>Example 2</u>. Solve: $x^2 - 27x + 126 = 0$.

Solution. Both roots are positive. Compose factor pairs of c = 126 with all positive numbers.
Proceeding: (1, 126)(2, 63)(3, 42)(6, 21). OK. This last sum is (6 + 21) = 27 = -b. Then, the 2 real roots are: 6 and 21.

TIP 3. When both real roots are negative (a and c same sign, a and b same sign), compose factor pairs of **c** with all negative numbers.

<u>Example 3</u>. Solve $x^2 + 31x + 108 = 0$

Solution. Both roots are negative. Compose factor pairs of c = 108 with all negative numbers. Proceeding: (-1, -108),(-2, -54),(-3, -36),(-4, -27). OK This last sum (-4, -27) = -31 = -b. Then the 2 real roots are: -4 and -27. There is no need for factoring and solving binomials.

CASE 2. When a ≠ 1 - Solving quadratic equations in standard form $ax^2 + bx + c = 0$ (1)

In this case, the "new AC Method" proceeds to transform the standard form into a simplified form $x^2 + bx + C = 0$, with a = 1, and with C = a*c. The transformed equation has this form: $x^2 + bx + a*c = 0$. (2)
Solving method for this transformed equation (2) goes back to CASE 1: finding 2 numbers x1 and x2 knowing their sum (-b) and their product (a*c).

Example 4. Solve: $8x^2 - 22x - 13 = 0$. (1) (AC = -13*8 = -104)

Solution. Solve the transformed equation: $x^2 - 22x - 104 = 0$ (2). Roots have opposite signs. Compose factor pairs of (-104) with all first numbers being negative.
Proceeding: (-1, 104),(-2, 52),(-4, 26). OK. This last sum is 22 = -b. Then, the 2 real roots of the equation (2) are: -4 and 26. Next, divide these numbers by a = 8 to get the 2 real roots of the original equation (1).
x1 = -4/8 = -1/2, and x2 = 26/8 = 13/4. No factoring by grouping.

Example 5. Solve: $12x^2 + 5x - 72 = 0$. (1) (AC = 12*-72 = -864)

Solution. Solve the transformed equation: $x^2 + 5x - 864 = 0$ (2). Roots have opposite signs. Compose factor pairs of -864 with all first numbers being negative. Since b = 5 is too small as compared to ac = -864, start composing factors from the middle:
Proceeding: …,(-16, 54),(-18, 48),(-24, 36),(-32, 27). OK. This sum is (-32 + 27) = -5 = -b.
The 2 real roots of (2) are: -32 and 27. Then, the 2 real roots of the original equation (1) are: x1 = -32/12 = -8/3, and x2 = 27/12 = 9/4. No factoring by grouping.

Example 6. Solve: $24x^2 + 60x + 36 = 0$. (AC = 24*36 = 864)

Solution. Solve transformed equation: $x^2 + 59x + 864 = 0$ (2). Both real roots are negative.
Compose factor pairs of a*c = 864 with all **negative numbers**, and start composing from the middle of the chain. Proceeding:…….(-9, -96),(-12, -72),(-16, -54),(-18, -48),(-24, -36), OK.
This sum (-24 - 36) = -60 = -b. The 2 real roots of (2) are: -24 and -36. Then, the 2 real roots of the original equation are: x1 = -24/24 = -1, and x2 = -36/24 = -3/2

Example 7. Solve: $16x^2 - 55x + 21 = 0$. (AC = 16*21 = 336)

Solution. Solve transformed equation: $x^2 - 55x + 336 = 0$ (2). Both roots are positive. Compose factors of ac with all numbers positive. Proceeding: (1, 336),(2, 168),(4, 82)(6, 56),(7, 48). OK.
This sum is 7 + 48 = 55 = -b. Then, the 2 real roots of (2) are: 7 and 48.
Back to the original equation (1), the 2 real roots are: x1 = 7/16, and x2 = 48/16 = 3.

More examples of solving by the New AC Method

Example 8. Solve $6x^2 + 17x - 14 = 0$ (1). (AC = -14*8 = -84)

Solution. Solve the transformed equation: $x^2 + 17x - 84 = 0$ (2). Roots have different signs. Compose factor pairs of a*c = -84 with all first numbers being negative. Proceeding: (-1, 84)(-2, 42)(-3, 28)(-4, 21). This last sum is 21 – 4 = 17 = b. Then, the 2 real roots are the opposite of -4 and 21. The 2 real roots are 4 and -21. Back to equation (1). The 2 real roots are: x1 = 4/6 = 2/3; and x2 = -21/6 = -7/2.

Example 9. Solve: $16x^2 - 62x + 21 = 0$ (1). (AC = 16*21 = 336)

Solution. Solve transformed equation $x^2 - 62x + 336 = 0$ (2). Both real roots are positive. Compose factor pairs of a*c = 336 with all positive numbers. Proceeding: (1, 336)(2, 168)(4, 82)(6, 56). The last sum is: 6 + 56 = 62 =-b. Then, the 2 real roots of (2) are: 6 and 56. Back to equation (1), the 2 real roots are; x1 = 6/16 = 3/8; and x2 = 56/16 = 7/2. No factoring.

Example 10. Solve: $12x^2 + 46x + 20 = 0$ (1). (AC = 16*21 = 240)

Solution. Solve transformed equation: $x^2 + 46x + 240$ (2). Both real roots are negative. Compose factor pairs of 240 with all negative numbers. Proceeding: (-1, -240)(-2, -120)(-3, -80)(-6, -40). The last sum is -46 = -b. Then, the 2 real roots of (2) are: -6 and -40. Back to the original equation (1), the 2 real roots are: x1 = -6/12 = -1/2; and x2 = -40/12 = -10/3.

CONCLUSION

The existing **factoring** "AC Method" can be considerably improved by applying the Rule of Signs for real roots into its solving approach. This improved method helps avoid the unnecessary lengthy steps of factoring by grouping and solving binomials. That is why this improved method may be simply called**: "The New AC Method".**
Its strong points are: simple, fast, no guessing, systematic, no factoring by grouping, and no solving binomials.

References:

- The new Transforming Method to solve quadratic equations (Google, Yahoo, Bing).
- The Bluma Method (Google, Yahoo)
- AC method for factoring. Regent Exam Prep Center at www.regentsprep.org
- Factoring trinomials – Simplified AC Method at 2000clicks.com/mathhelp

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