THE NEW AC METHOD TO FACTOR TRINOMIALS.

(authored by Nghi H Nguyen – March 2015)

A trinomial in x has as standard form: $f(x) = ax^2 + bx + c$. Factoring this trinomial means transforming it into two binomials in x.

A. WHEN a = 1 – FACTORING TRINOMIAL TYPE: $f(x) = x^2 + bx + c$

The factored trinomial will have the form: f(x) = (x – p)(x – q). The values of p and q can be computed from the values of the coefficients a, b, and the constant c.

The New AC Method proceeds to find two numbers p and q that satisfy these 3 conditions:

1), The product p*q = a*c. (when a = 1, this product is c).

2). The sum (p + q) = b

3). Application of the Rule of Signs.

RECALL THE RULE OF SIGNS.

- When a and c have different signs, p and q have different signs
- When a and c have same sign, p and q have the same sign

OPERATIONAL METHOD. To find p and q, we compose factor pairs of c, and, in the same time, apply the Rule of Signs. The pair whose sum equals to (b), or (-b,) gives p and q.

<u>Example 1</u>. Factor: $f(x) = x^2 - 11x - 102$. (a*c = c = -102)

Solution. The numbers p and q have different signs (since a and c have different signs). Compose factor pairs of (c = -102) with all first numbers being negative. Proceed: (-1, 102)(-2, 51)(-3, 34)(-6, 17). The last sum is: (-6 + 17) = 11 = -b. By changing to the opposite (-b → b), we then get: p = 6 and q = -17. The factoring form is:

f(x) = (x + 6)(x – 17). No need to factor by grouping.

<u>Example 2</u>. Factor: $f(x) = x^2 - 28x + 96$.

Solution. Since a and c have same sign, p and q have same sign. Compose factor pairs of (c = 96) with all positive numbers. Proceed: (1, 96)(2, 48)(3, 32)(4, 24). This last sum is (4 + 24) = 28 = -b. Then, p = -4 and q = -24. The factoring form is:

f(x) = (x – 4)(x – 24). No factoring by grouping.

Example 3. Factor: $x^2 + 31x + 108$.

Solution. p and q have same sign. Compose factor pairs of (c = 108) with all positive numbers. Proceed: (1, 108)(2, 54)(3, 36)(4, 27). This last sum is (4 + 27) = 31 = b. Then, p = 4 and q = 27. The factoring form is: f(x) = (x + 4)(x + 27).

B. WHEN a ≠ 1 – FACTORING STANDARD TRINOMIAL TYPE: $ax^2 + bx + c$ (1)

To factor this trinomial type, we bring it back to **Case A** (a = 1). Convert this trinomial f(x) to the trinomial $f'(x) = x^2 + bx + a*c$ (**2**), with a = 1, and the constant is (a*c). Then, we proceed finding 2 numbers p' and q' like we did in **Case A**. Next, we divide p' and q' by **a** to get p and q by the relations: p = p'/a, and q = q'/a.

Example 4. Factor: $f(x) = 8x^2 - 22x – 13$ (1). (a*c = 8*-13 = -104)

Solution. Convert f(x) to $f'(x) = x^2 - 22x – 104$ (2). Find the 2 numbers p' and q'. The numbers p' and q' have different signs. Compose factor pairs of (a*c = -104). Proceed: (-1, 104)(-2, 52)(-4, 26). This last sum is: (-4 + 26) = 22 = -b. By changing to the opposite (b), we get: p' = 4 and q' = -26. Then, p = p'/a = 4/8 = 1/2; and q = q'/a = -26/8 = -13/4. The factoring form of original f(x) will be: f(x) = (x + 1/2)(x – 13/4).

Finally, f(x) = (2x + 1)(4x – 13). No factoring by grouping.

Example 5. Factor: $f(x) = 15x^2 - 53x + 16$. (a*c = 15*16 = 240)

Solution. Converted trinomial $f'(x) = x^2 - 53x + 240$ (2). The 2 numbers p' and q' have same sign. Compose factor pairs of (a*c = 240) with all positive numbers. Proceed: (1, 240)(2, 120)(3, 80)(4, 60)(5, 48). This last sum is (5 + 48) = 53 = -b. Then, p' = -5 and q' = -48. Next, p = p'/a = -5/15 = -1/3; and q = q'/a = -48/15 = -16/5. The factoring form of f(x) will be: f(x) = (x – 5/15)(x – 48/15) = (x – 1/3)(x - 16/5). Finally, f(x) = (3x – 1)(5x – 16).

Example 6. Factor: $f(x) = 12x^2 + 83x + 20$. (1) (a*c = 12*20 = 240)

Solution. Converted trinomial $f'(x) = x^2 + 83x + 240$ (2). The numbers p' and q' have same sign. Compose factor pairs of (a*c = 240). Proceed: (1, 240)(2, 120)(3, 80). This last sum is (3 + 80) = 83 = b. Then, p' = 3 and q' = 80. Back to original f(x), p = 3/12 = 1/4; and q = 80/12 = 20/3. The factoring form is: f(x) = (x + 1/4)(x + 20/3). Finally, f(x) = (4x + 1)(3x + 20).

NOTE 1. When composing factor pairs of (a*c), or c, if we can't find the pair whose sum equals to (b), or –b, then this trinomial can't be factored.

NOTE 2. We don't need to factor the converted trinomial f'(x). We just need to find the 2 numbers p' and q' in order to get the 2 numbers p and q for the original trinomial f(x).

MORE EXAMPLES OF FACTORING BY THE NEW AC METHOD.

Example 7. Factor: $6x^2 + 17x – 14$. (1) (a*c = 6*-14 = -84)

Solution. Converted trinomial $f'(x) = x^2 + 17x – 84$ (2). The numbers p' and q' have different signs. Compose factor pairs of (a*c = - 84). Proceed: (-1, 84)(-3, 41)(-4, 21). This last sum is (-4 + 21) = 17 = b. Then, p' = -4 and q' = 21. Back to original f(x). The factoring form is: f(x) = (x - 4/6)(x + 26/6) = (x - 2/3)(x + 7/2). Finally, f(x) = (3x - 2)(2x + 7).

Example 8. Factor: $16x^2 - 55x + 21$. (1) (a*c = 16*21 = 336)

Solution. Converted trinomial $f'(x) = x^2 - 55x + 336$ (2). Compose factor pair of (a*c = 336) with all positive numbers. Proceed: (1, 336)(2, 168)(4, 82)(6, 56)(7, 48). This last sum is (7 + 48) = 55 = –b. Then p' = -7 and q' = -48. Back to trinomial (1), p = p'/a = -7/16; and q = q'/a = -48/16 = -3. Finally, the factoring form of (1) is: f(x) = (16x – 7)(x – 3).

Example 9. Factor: $12x^2 + 46x + 20$ (1). (a*c = 12*20 = 240)

Solution. Converted trinomial $f'(x) = x^2 + 46x + 240$ (2). The 2 numbers p' and q' have same sign. Compose factor pairs of (a*c = 240). Proceed: (1, 240)(2, 120)(3, 80)(6, 40). This last sum is (6 + 40) = 46 = b. Then, p' = 6, and q' = 40. Back to original trinomial (1): p = p'/a = 6/12 = 1/2, and q = q'/a = 40/12 = 10/3. Finally, the factoring form is f(x) = (2x + 1)(3x + 10).

CONCLUSION. The strong points of this New AC Method to factor trinomials are: simple, fast, systematic, no guessing, no lengthy factoring by grouping.

[This article was written by Nghi H Nguyen, author of the New AC Method to solve quadratic equations (Yahoo, Bing, and Google Search) – March 2015]