A FEW APPLICATIONS ON THE CALCULATIONS OF SOME LIMITS

An application refers to the calculation of lateral limits. Specifically, in order to calculate a lateral limit according to the proposed method, an auxiliary variant ε of a very small value is introduced, we can say that it tends to zero in this context. This very small value makes the differentiation between the right lateral limit and the left lateral limit.

We will describe the method by examples.

So if we must verify the continuity of the function $f(x) = \frac{2x-1}{2\sqrt{x^2-x}}$ in point x=1, after the classical method, we will calculate the lateral limits as follows:

$$\mathrm{ld}_{\substack{x\to 1\\ x\searrow 1}} f(x) = \lim \mathrm{d}_{x\searrow 1} \frac{2x-1}{2\sqrt{x^2-x}} = \infty \,;$$

$$\mathrm{ls}_{\substack{x\to 1\\ x\nearrow 1}} f(x) = \lim \mathrm{s}_{x\nearrow 1} \frac{2x-1}{2\sqrt{x^2-x}} = -\infty$$

After the method proposed in this article, these limits are differentiated by introducing the variable ε with a very small value as follows:

$$\mathrm{ld}_{\substack{x\to 1\\ x=1+\varepsilon\\ \varepsilon\ll}} f(x) = \lim_{\substack{x=1+\varepsilon\\ \varepsilon\ll}} \frac{2x-1}{2\sqrt{x^2-x}} = \lim_{\varepsilon\to 0} \frac{2(1+\varepsilon)-1}{2\sqrt{\varepsilon^2+\varepsilon}} = \infty \,;$$

$$\mathrm{ls}_{\substack{x\to 1\\ x=1-\varepsilon\\ \varepsilon\ll}} f(x) = \lim_{\substack{x=1-\varepsilon\\ \varepsilon\ll}} \frac{2x-1}{2\sqrt{x^2-x}} = \lim_{\varepsilon\to 0} \frac{-2\varepsilon+1}{2\sqrt{\varepsilon^2-\varepsilon}} = -\infty \,;$$

Thus, the calculation of lateral limits is easier to apply. Another example is the calculation of the limits for x tending to a negative value, x usually tends to $-\infty$ as in the example below:

$$\lim_{x\to-\infty} \frac{e^x}{x+1}$$

In this case it is more advisable to use an auxiliary variable $t = -x$, which would tend to $+\infty$, namely :

$$\lim_{t\to\infty} \frac{e^{-t}}{-t+1} = \lim_{t\to\infty} \frac{1}{e^t(-t+1)}$$ which obviously tends to zero.

Of course that by the material presented we gave examples of proposed methods, but these methods are not restrictive and can be extended to other cases, as they are practical methods with accurate and immediate results.

Author

Adrian Stanculescu, Engineer, Ph.D.